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AGING AND PUBLIC PENSIONS IN AN OVERLAPPING-GENERATIONS MODEL

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In this paper a social-welfare maximizing public-pension policy is modeled within the framework of the well-known two-overlapping-generations general-equilibrium model with rational expectations. The model is used to analyze the effects of aging on the evolution of public pension schemes. Analytical results are derived for the long run as well as for the short run by the method of comparative statics and comparative dynamics respectively. This shows that the short-run consequences of aging depend crucially on the existing size of the PAYG-scheme.

1. Introduction

IN A path-breaking paper Cutler *et al.* (1990) asserted that in a command economy an optimal policy response to aging could be to decrease the national saving rate and increase consumption. Using numerical simulations they showed that if aging is not too severe in the next two decades (as in the US) short-run consumption gains can be obtained while the current generations will be worse off if the aging of the population is beginning to be felt immediately (as in the case of Japan). They derive their results in a Ramsey-model of economic growth where a central planner maximizes a Benthamite social welfare function (SWF).

The drawback of the Ramsey model is that agents are supposed to live infinitely long so that, as a result, intergenerational redistribution effects of aging and the consequences of these effects for savings cannot be taken into account.¹ Therefore, our paper considers this issue in the framework of an overlapping-generations (OLG) model. While sticking to the Benthamite SWF,² intergenerational transfers are explicitly modeled by a Pay-as-You-Go (PAYG) public pension scheme. Our paper shares the objectives of Bovenberg *et al.* (1993) and Auerbach and Kotlikoff (1987). However, unlike our paper they are not able to give analytical solutions for transitions paths and have to rely on numerical solutions.

The closed-economy general-equilibrium framework used in this paper

¹ Note that, contrary to what is suggested by Cutler *et al.*, the two-step procedure used in Calvo and Obstfeld (1988) is not applicable in case of aging and a Benthamite SWF.

² As is well known, with a Benthamite SWF the steady-state capital-labor ratio does not depend on the size of the population, contrary to a more conventional, so-called Millian, SWF where utility is not weighted by population size. However, the choice of the SWF is not the issue here. Ethical (Blanchet and Kessler, 1991) or political (Meijdam and Verbon, 1996) arguments may be brought to the fore to justify the choice of the Benthamite SWF. In fact, we have chosen the Benthamite SWF to make our results comparable to those of Cutler *et al.* However, our method can be applied to a Millian SWF as well.

is presented in Section 2. In this section, a central planner is introduced that uses PAYG-transfers as an instrument to realize an optimal allocation of production and consumption to young and old individuals, respectively. In Section 3 we investigate how this optimal PAYG-pension system reacts to an aging of the population such as can be expected to occur in the next century in many countries. Aging implies, first, due to the increase of the dependency ratio, that total output per worker has to be shared with a larger number of pensioners, leading to lower PAYG-benefits (dependency-ratio effect). Secondly, aging implies that less saving is necessary to maintain the (unchanged) steady-state capital-labor ratio leading to higher consumption possibilities for both young and old individuals (capital-thickening effect). It is trivial to demonstrate that in the steady state the former effect dominates the latter effect. So, the long-run effect of aging is to decrease individual utility. Short-run effects are, however, less easily obtained. We derive analytical results for the short run for fairly general specifications for utility and production functions. It appears that the short-run effects can differ considerably from the long-run effects. More specific, we are able to qualify the above mentioned result suggested by Cutler *et al.*, namely that an expected aging of the population will lead to short-run gains in consumption possibilities whereas an unexpected aging decreases consumption. It will be demonstrated that the occurrence of short-run gains depends not so much on the fact whether aging is expected or not but on the size of the existing PAYG-scheme. In particular, if this size is relatively small, according to the Benthamite SWF, aging should initially lead to rising consumption possibilities for young as well as old individuals under both an expected and an unexpected decrease in population growth. This can be accomplished by the government in a decentralized economy through the extension (or an introduction) of the PAYG-scheme. This way, it smooths utility between generations living at the same time. In this closed-economy model, however, it does not perfectly smooth consumption over time as the intertemporal allocation of consumption is determined by a discrete-time version of the Euler equation that is well-known from the Ramsey model. In fact, the initial increase in consumption possibilities realized by the social planner goes at the cost of a decrease in utility for all future generations. The paper ends with some concluding remarks in Section 4.

2. The model

Each period a new generation consisting of a large number of identical individuals gifted with perfect foresight and living for two periods is born. So at each point in time there are two generations: the young and the old. Each individual takes aggregate savings, wage rates, interest rates, and tax rates as given. An individual born at time t works a fixed amount of time at wage w_t during the first period of his life. A part of this wage is taxed away by the government by a lump-sum tax τ_t to be transferred to the old. The remainder is used for savings for old age (s_t) and for consumption: $c_t^y = w_t - \tau_t - s_t$.

When old, the individual consumes the return on his savings and the transfer payment η from the government. So the consumption at time $t + 1$ of an old individual born at time t is equal to $c_t^r = (1 + r_{t+1})s_t + \eta_{t+1}$. Individuals are assumed not to be altruistic: they do not care about the levels of utility of individuals born in another time period. It is assumed that lifetime utility of an individual born at time t can be represented by a separable utility function $U_t = u(c_t^y) + (1/(1 + \theta))u(c_t^r)$, where the felicity functions obey the Inada-conditions. The parameter θ indicates the rate of time preference.

Production per worker at time t is described by a standard neoclassical production function

$$f(k_t), \left(f'(k) \equiv \frac{df}{dk} > 0, f'' \equiv \frac{d^2f}{dk^2} < 0 \right)$$

where k_t is the capital-labor ratio. Abstracting from depreciation and assuming that firms act competitively and that labor supply is exogenous, factor market equilibrium is described by the marginal productivity conditions $f'(k_t) - k_t f''(k_t) = w_t$ and

$$f'(k_t) = r_t \quad (1)$$

Equilibrium on the goods market implies for this closed-economy model that investment equals savings or in per worker terms

$$(1 + n_{t+1})k_{t+1} = s_t \quad (2)$$

where n_t is the rate of population growth at time t which is assumed to be determined exogenously. The following resource constraint for the economy as a whole follows from the above definitions

$$f(k_t) + k_t = (1 + n_{t+1})k_{t+1} + c_t^y + \frac{1}{1 + n_t} c_{t-1}^r \quad (3)$$

which simply says that total supply of goods can be allocated to investment or to consumption by the currently living generations.

The government at time t is assumed to maximize a Benthamite social welfare function with a social discount rate ρ

$$W_t = \sum_{i=t}^{\infty} U_{i-1} \prod_{j=t}^i \frac{1 + n_{j-1}}{1 + \rho}, \quad (0 < \rho < \infty) \quad (4)$$

According to eq. (4), the government is a social planner who takes account of the welfare of all current and future generations where the welfare of a generation is measured by the utility of a representative individual of this generation weighted by the size of the generation.

Assuming that $n_t < \rho$, the first-order conditions for a command optimum that maximizes this social welfare function can easily be derived

$$u'(c_{t-1}^r) = \frac{1 + \theta}{1 + \rho} u'(c_t^y) \quad (5)$$

$$u'(c_{t-1}^y) = \frac{1 + f'(k_t)}{1 + \rho} u'(c_t^y) \quad (6)$$

where $u'(c) \equiv (du/dc)$. Equation (5) is a condition for optimal allocation to young and old individuals alive at the same time and eq. (6) optimizes intertemporal allocation. This equation is a discrete-time equivalent of the Euler equation that is familiar from the Ramsey model with the social discount rate instead of the private discount rate. It is well known that this command optimum can be replicated in a market economy using intergenerational redistribution through a PAYG-scheme (see Blanchard and Fischer, 1989, Section 3.1). In particular, combining eqs (1), (5), and (6) would lead to the first-order condition for individual saving. So, the central planner respects individuals' savings decisions. Abstracting from administrative costs, the budget restriction of a PAYG-scheme reads $\eta_t = (1 + n_t)\tau_t$.

It follows directly from eqs (1) and (6) that the steady state of the model is characterized by³

$$r = \rho \quad (7)$$

This is the well-known modified golden rule: the higher the social discount factor, the lower the capital stock per worker in the steady state will be, which is, of course accomplished by a higher transfer which has a negative effect on savings. Notice that, as stated in the Introduction, the steady-state capital-labor ratio is independent of population growth n and that the assumption $\rho > n$ implies that the steady state of the economy is dynamically efficient.

3. The effects of aging

Given a government fixing tax rates in an optimal way as prescribed by eqs (5) and (6), it is of interest to consider the effects of aging reflected by a decrease in population growth (n). In this section the short-run as well as the long-run consequences of once-and-for-all changes in n are derived analytically. Most results can directly be generalized to temporary changes in this parameter. Medium-term effects can be analyzed analytically as well, but we restrict ourselves here to an illustrative numerical example. The long-run effects follow from comparing steady states, while the short-run consequences are traced by

³ When no time subscript is used the steady-state value is meant.

comparative dynamics.⁴ Regarding the short run, only the initial effects, i.e. the effects at $t = 0$, are considered. The effects of unexpected and expected shocks are distinguished. This distinction can be motivated by the fact that the timing of the aging processes differs dramatically in the developed world. In the US e.g., aging will be felt in its consequences not before 2010, while Japan is currently being confronted with the consequences of aging. So, the analysis of an expected shock is applicable to the US, while for Japan unexpected shocks are relevant.

3.1. Short-run effects

The comparative dynamics analysis is based on linearization of the model presented above around the steady state. The analysis concentrates on the effects of once-and-for-all decreases in population growth (n). Let $n_t = n + \gamma h_t$, where h_t describes the time pattern of the perturbation of the steady-state value of the parameter n and γ the magnitude. It is assumed that at some time $t = 0$ a shock unexpectedly occurs (i.e. $h_0 = h_1 = \dots = h < 0$) or is expected to occur at time $z > 0$ (i.e. $h_0 = h_1 = \dots = h_{z-1} = 0$, $h_z = h_{z+1} = \dots = h < 0$). It will be convenient in the sequel to introduce total consumption per worker which is defined as

$$c_t = c_t^y + \frac{c_{t-1}^r}{1 + n_t} \quad (8)$$

As described in the Appendix, the linearized model can be condensed to a system describing the changes in this variable and in the capital-labor ratio k_t

$$\begin{bmatrix} \frac{\partial k_{t+1}}{\partial \gamma} \\ \frac{\partial c_{t+1}}{\partial \gamma} \end{bmatrix} = J \begin{bmatrix} \frac{\partial k_t}{\partial \gamma} \\ \frac{\partial c_t}{\partial \gamma} \end{bmatrix} + M \begin{bmatrix} h_t \\ h_{t+1} \end{bmatrix}, \quad t = 0, 1, \dots \quad (9)$$

where M is a matrix describing the effects of the current and next-period change in n on the state variables and J is the Jacobian matrix defined in the Appendix. As the system in eq. (9) is saddlepoint stable, it can be solved to find the short-run effect of changes in n on total consumption per worker, i.e. $(\partial c_0 / \partial \gamma)$ (see Blanchard and Kahn, 1980) which is given by:

$$\begin{aligned} \frac{\partial c_0}{\partial \gamma} = & \frac{\lambda_2 - 1}{\lambda_2} \left(\frac{c^r}{(1+n)^2} - \lambda_2 k \right) \sum_{j=1}^{\infty} h_j \lambda_2^{-j} \\ & - (1+n)(\delta + 1 - \lambda_2) \frac{\partial k_0}{\partial \gamma} - \frac{1}{\lambda_2} \frac{c^r}{(1+n)^2} h_0 \end{aligned} \quad (10)$$

⁴ This method is described by Judd (1982) for a continuous-time model. It can easily be generalized to discrete time, however. See Meijdam and Verhoeven (1994), who also show that the approximation of the short-run effects of a parameter change by this method is as reliable as a standard comparative-statics analysis. The method is explained in the Appendix.

where $\delta(>0)$ is a constant defined in the Appendix and $\lambda_2(>1+\delta)$ is the unstable root of the linearized system.

Notice from eq. (10) that the optimal response of total consumption per worker is in part determined by future changes in the rate of population growth. This is a reflection of the forward-looking behavior of the central planner who takes account of the utility of all future generations. So eq. (10) is the result of the fact that the government, when observing a permanent shock, redistributes income among all current and future generations by varying the PAYG tax rate in accordance with its objective function (4). Note that eq. (10) might be dubbed a short-run macroeconomic effect as it gives the additional (positive or negative) consumption (expressed per worker) available for the whole economy at $t=0$. This additional consumption has to be allocated to the currently living young and old individuals.

The short-run macroeconomic effect on consumption per worker appears to consist of several elements. Firstly, the initial effect on total consumption per worker is in part determined by anticipation of the future effects of the change in n as reflected by the appearance of the terms $h_j(j>0)$ in eq. (10). Two anticipation effects can be discerned. The first is the capital-thickening effect (see the term $\lambda_2 k$ in eq. (10)).⁵ This effect stems from the fact that a decrease in n implies that *ceteris paribus* a lower level of saving per worker is necessary in order to realize a certain level of the capital-labor ratio next period. So, the capital-thickening effect implies an immediate decrease of the saving rate, leading to an increase of the consumption per worker. The second anticipation effect comes from the fact that an increase in the so-called dependency ratio, defined as the number of pensioners per worker, implies that total consumption per worker has to be shared with a larger number of pensioners. This dependency-ratio effect ($c'/(1+n)^2$) decreases total consumption per worker.

In case of an unexpected decrease in n , there are not only anticipation effects, but also immediate effects on total consumption per worker at $t=0$. The dependency ratio, for example, increases immediately. However, if the change in population growth is once-and-for-all, so that $h_j = h \forall j \geq 0$, then the initial dependency-ratio effect exactly compensates the future effect. Moreover, an unexpected decrease in n causes an upward jump in the capital-labor ratio, which implies a higher production and thus a higher consumption per worker. This is reflected by the term containing $(\partial k_0 / \partial \gamma) = -(k/(1+n))h_0 > 0$ in eq. (10). It immediately follows that the short-run macroeconomic effect on total consumption per worker in the case of an unexpected once-and-for-all shock is positive.

In case of an expected decrease in n , the anticipation effects are the only short-run effects on total consumption per worker. Whether total consumption per worker will increase then depends on the difference between the dependency-

⁵ This is exactly the opposite of the capital dilution effect that occurs in case of an increase in n . See e.g., Blanchet and Kessler (1991, p.140).

ratio effect and the capital-thickening effect, as can be seen from the first line of eq. (10).

Total consumption per worker has to be shared by young and old individuals, respectively, according to eq. (8). Differentiating eq. (8) and using the first-order conditions (5) and (6) we get

$$\frac{\partial c_t^y}{\partial \gamma} = (1 - \varepsilon) \left[\frac{\partial c_t}{\partial \gamma} + \frac{c^r}{(1 + n)^2} h_t \right] \quad (11)$$

$$\frac{\partial c_{t-1}^r}{\partial \gamma} = (1 + n) \varepsilon \left[\frac{\partial c_t}{\partial \gamma} + \frac{c^r}{(1 + n)^2} h_t \right]. \quad (12)$$

where ε is defined in the Appendix. It should be noted that in the case of an expected shock $h_0 = 0$ and the change in individual consumption levels c_0^y and c_{-1}^r is proportional to the change in total consumption per worker, i.e. $(\partial c_0 / \partial \gamma)$. On the other hand, if the shock is unexpected $h_0 < 0$ and the positive macro-economic effect $(\partial c_0 / \partial \gamma) > 0$ can be dominated by the dependency-ratio effect dependent upon the level of consumption by the old in the steady state.

So, for both types of shocks individual consumption may fall or rise in the short-run. This seems in line with the results of Cutler *et al.* (1990) who conclude that decreases in the rate of population growth have theoretically ambiguous initial effects on individual consumption. However, our analysis qualifies this ambiguity as it turns out that whether consumption rises or falls depends to a large extent on the size of the PAYG-scheme measured by the tax rate. In particular, using eqs (10), (11), and (12), the following proposition can easily be derived:

Proposition 1. An unexpected once-and-for-all decrease in n initially decreases individual consumption for young and old individuals iff $\tau > (1 - \lambda_1)s$. An expected once-and-for-all decrease in n initially decreases individual consumption for young and old individuals iff $\tau > (1 - \lambda_1 + \delta)s$.⁶

The interpretation of this proposition is straightforward: If, in the initial steady state, the intergenerational transfers (as measured by the tax rate τ) are high compared with individual saving, then the increase in the consumption possibilities per worker due to the decrease in savings of the young is not sufficient to provide the relatively larger number of elderly per worker with the same level of consumption as before the shock. The intuition is that if the tax rate is relatively large, old-age consumption is relatively large. Consequently, large increases in total consumption per worker are necessary in order to be able to maintain the consumption level of the increased number of elderly without harming the young. However, a high tax rate goes along with low savings, implying a low capital-labor ratio and therefore relatively low consumption gains per worker due to the capital-thickening effect. Hence, the

⁶ $\lambda_1 < 1$ is the stable root of the linearized system.

government will choose to decrease individual consumption for both generations. This is achieved by adjusting the tax rate.⁷

Comparing an expected with an unexpected shock, it is of interest to observe that no qualitative difference exists regarding the initial effects. There is, however, a difference of a quantitative nature as can be seen from the appearance of the parameter δ in Proposition 1. This (positive) parameter can be interpreted as a measure for the speed at which changes in consumption and investment are transmitted to future periods and vice versa. In particular, if δ has a low value, anticipated future shocks will strongly affect current values and expected and unexpected shocks will have similar results.

The effect of a change in n on the tax rate in period t is given by

$$\frac{\partial \tau_t}{\partial \gamma} = [rv - 1 - r] \frac{\partial k_t}{\partial \gamma} + \varepsilon \frac{\partial c_t}{\partial \gamma} - (1 - \varepsilon) \frac{c^r}{(1 + n)^2} h_t \quad (13)$$

The first term in this equation indicates the effect of a change in the capital–labor ratio. At a given interest rate, an increase in the capital stock per worker implies an increase of the old-age consumption per worker ($c^r/(1 + n)$) of $(1 + r)(\partial k_t/\partial \gamma)$, thus decreasing the need for redistribution from the young to the elderly through τ . An increase in the capital–labor ratio also decreases the interest rate and raises the wage rate, thus increasing the need for redistribution. This effect is measured by v , which is the (positively defined) elasticity of the production function $v = (-f''(k)k/f'(k))$. The larger this elasticity, the larger the effect of a change in the capital–labor ratio on the interest rate. The second term in equation (13) represents the effect that, as a part ε of total consumption per worker goes to the old, more total consumption per worker *ceteris paribus* increases the tax rate. In the third term the dependency-ratio effect can be recognized again. An increase in the dependency ratio *ceteris paribus* increases the tax rate. So, whether it will be necessary to increase or decrease the tax rate in case of an unexpected change in the rate of population growth, depends on the strength of the general-equilibrium effects caused by the initial increase of the capital–labor ratio that takes place in that case. If the decrease in n is expected, the capital–labor ratio and the dependency ratio do not change initially. Then the following corollary follows from eq. (13) and Proposition 1:

Corollary 1 An expected once-and-for-all decrease in n initially increases the tax rate iff $\tau \leq (1 - \lambda_1 + \delta)s$.

Obviously, as consumption of the old at $t = 0$ can only increase if the tax rate is raised, this corollary is just a rephrasing of Proposition 1. It has, however,

⁷ Whether the tax rate is indeed large enough for a decrease in population growth to decrease individual consumption in the short run turns out to be an empirical question. For Cobb–Douglas production and utility functions and reasonable parameter values ($n = 0.35$ (when a period is taken to be 30 years, this corresponds to a population growth of 1% per year), $\rho = 3.3$ (5% per year), $\theta = 1.8$ (3.5% per year), capital share = 0.33), the tax rate (8% of GNP) is slightly above the critical level for an unexpected shock (7.5% of GNP). Decreases in ρ and increases in θ , n , and the share of capital decrease the tax rate relative to its critical level.

an interesting implication. It implies that in the absence of a PAYG-financed public pension system (i.e., $\tau = 0$) such a system can come about in anticipation of a decrease in the rate of population growth. This result is rather surprising as intuition might suggest that an increasing population would give more room for the installation of a public pension system due to the low tax rates implied by high rates of population growth. The point is that an affluent capital stock at the time of the shock is anticipated by the forward-looking government. This incites it to increase the tax rate, simultaneously leading to a decrease of the savings by all generations living before the shock. These desinvestments lead to an increase in the consumption possibilities for old and young individuals. Some remarks on this result will be made in the concluding section.

Another implication of the corollary is that in countries where the aging of the population will not occur before the next century but where the PAYG-system is relatively extensive it may be necessary nevertheless to immediately cut in intergenerational transfers.

3.2. *The long-run effects of a decrease in n*

In the long run, the capital stock per worker remains unchanged as immediately can be seen from the modified golden rule (eq. (7)). Moreover, the redistribution effects of a change in the dependency ratio cancel out and do not affect total consumption. So the only effect on total consumption per worker in the long run is the positive capital-thickening effect. Consequently, total consumption per worker will be higher, as can be seen from

$$\frac{\partial c}{\partial \gamma} = -\frac{\partial s}{\partial \gamma} = -\frac{s}{1+n} h > 0 \quad (14)$$

As in Cutler *et al.*, the long-run effect of aging on individual consumption is the sum of a positive capital-thickening effect and a negative dependency-ratio effect. In contrast to that paper, here the net effect can easily be determined as there is an explicit relation between these two effects. Using eqs (14) and (11) or (12), it immediately follows that the capital-thickening effect on individual consumption outweighs the dependency-ratio effect iff the part of total consumption per worker consumed by pensioners ($c^r/(1+n)$) is smaller than savings per worker, which is equivalent to $\tau < (n-r)k$. The fact that the economy is dynamically efficient then immediately leads to the following proposition:

Proposition 2 If the tax rate is non-negative, a decrease in n decreases individual consumption in the long run.

As in the long run the capital-labor ratio does not change, it can easily be derived that the long-run tax rate is a negative function of the rate of population growth. Consequently, the effect of a decrease in population growth on the long-run transfer payment η is positive if $\tau \leq 0$. If the tax rate is positive, however, the effect on the transfer payment is ambiguous.

3.3. Medium-term effects

The evolution of c and k after the initial change at $t = 0$ can be traced by using system (9). The changes in all other variables can be derived from the changes of these state variables. The analytical derivations do not produce any additional insight, however. Instead, we produce a numerical example in order to illustrate the mechanics of the model. Suppose that the economy is in the steady state when at time $t = 0$ a permanent decrease in the rate of population growth is expected to occur three generations later. The most important effects of this shock are reproduced in Figs 1a, 1b, and 1c.⁸

As for this example $\tau < 1 - \lambda_1 + \delta$ holds, consumption for young and old individuals increases at $t = 0$ according to Proposition 1. However, in the long-run individual consumption will be below the previous steady-state value (see Figure 1a). The figures nicely bring out what the reasons are for the divergence of short-run and long-run effects. Without government intervention, at the time of the shock a large capital abundance would appear which, in this closed economy, would strongly decrease the interest rate. Consequently, old-age consumption of the generation born at $t = 2$ would decrease and consumption possibilities for later generations would increase. The timepath of lifetime utility that would result in this case without PAYG can be found in Figure 1c. This timepath is not optimal according to the Benthamite SWF, however. As can be seen from eq. (6), the optimal intertemporal allocation of consumption depends on the difference between the interest rate and the social

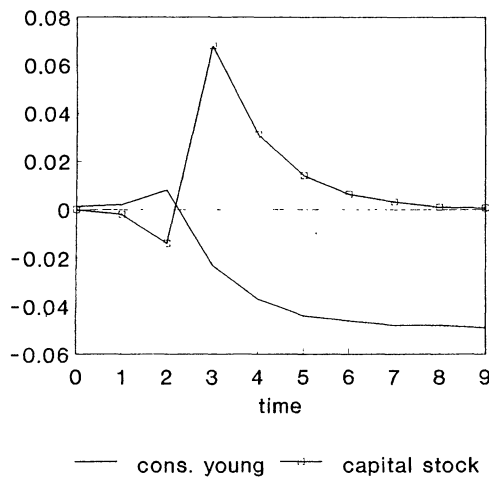
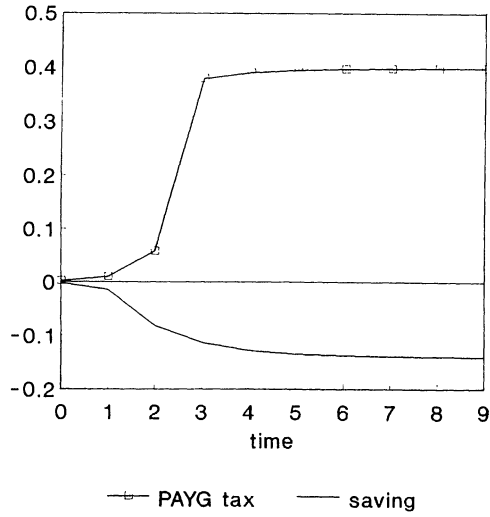
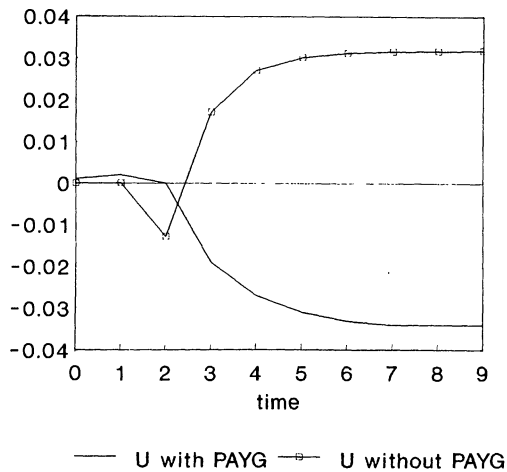


FIG. 1a. The effect of an expected permanent decrease in n on consumption and capital

⁸ We used the specification of the model described in footnote 7, except for the social discount rate where we use a lower value ($\rho = 2.25$) such that there is no PAYG-scheme (i.e., $\tau = 0$) in the initial steady state. The impulse is a decrease in population growth of 0.5% ($n = 0.16$) from $t = 3$ on. Except for τ , the figure presents changes in the variables relative to their steady state values.

FIG. 1b. The effect of an expected permanent decrease in n on savings and taxesFIG. 1c. The effect of an expected permanent decrease in n on utility

discount rate. Therefore, as can be seen from Figure 1b, the social planner will partly prevent the capital abundance by decreasing savings through increasing the tax rate in all previous periods. As from $t = 3$, the tax rate increases strongly due to the dependency-ratio effect. As a result of this government intervention, generations born at the time of the shock and thereafter lose, while generations living before the shock gain (see Figure 1c). So, the central planner, while smoothing the utility between both old and young generations living at the

same time, will not smooth utility between different time periods. On the contrary, compared with a world without social planner (i.e., $\tau = 0$), the gains from aging for future generations are almost completely turned into losses.

4. Concluding remarks

In this paper the effects of aging on optimal PAYG-financed public pension schemes and individual consumption have been analyzed in an overlapping-generations general-equilibrium framework. The long-run effects of aging are well-known. This knowledge is summarized in Proposition 2 of this paper. This proposition says that in the long run the capital-thickening effect is dominated by the dependency-ratio effect, leading to lower consumption for both young and old individuals. The short-run effects of aging, however, are much less known in the literature. In particular, Cutler *et al.* (1990) claim these effects to be ambiguous, but, at the same time, suggest by numerical simulations that an unexpected decrease in the rate of population growth will have a negative effect on individual consumption, while if this decrease is expected to occur after a few decades, initial gains in consumption will be obtained.

This paper has derived the initial effects of aging analytically. The results are reflected in Proposition 1. This proposition claims that for unexpected as well for expected aging of the population the size of the PAYG-financed public pension scheme is the determining variable for the initial effects. In particular, for an extensive public pension system both types of shocks may lead to losses in individual consumption. On the other hand, if the PAYG-taxes are relatively small, gains in consumption made possible by the lower need for savings will not be dominated by the requirement for higher transfers to maintain the level of old-age consumption. If the young then experience a consumption gain, this will, in an overlapping-generations model, partly have to be transferred to the old through an increase in the tax rate. So, it follows that if a PAYG-scheme is non-existent, it can come about because of an expected decrease in the rate of population growth. With an eye turned to reality, this might suggest that the abundant growth in coverage and size of public pension schemes which was manifest in the Western world during the first two decades after World War II may, in face of the anticipation of the aging of the population, have increased social welfare. Recent measures taken in the Western world aimed at restricting the growth rate of the taxes could then be explained from the fact that demographic changes appear to be much more dramatic than expected. In particular, given the then existing extensive PAYG-scheme, the sharp decrease in birth rates that started in the seventies in the EC countries made a drop in benefit rates unavoidable. Obviously one should be cautious in deriving policy conclusions from the stylized model used in this paper. The two-OLG set-up, used here for analytical convenience, may overstate the life-cycle effect of savings. Moreover, the assumption that public pensions are financed by lump-sum taxes is not very realistic. A straightforward extension of the analysis would be to allow for taxes on wage income and endogenous labor supply.

Finally, more research is needed in order to establish whether the assumption that the government acts as a central planner maximizing a Benthamite SWF is a reasonable approximation of actual public-pension policy.

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APPENDIX

Equation (9) is derived by taking a first-order Taylor expansion of eqs (3), (5), and (6) around the steady state given by eq. (7). This gives, respectively

$$(1+r) \frac{\partial k_t}{\partial \gamma} = (1+n) \frac{\partial k_{t+1}}{\partial \gamma} + k h_{t+1} + \frac{\partial c_t^y}{\partial \gamma} + \frac{1}{1+n} \frac{\partial c_{t-1}^r}{\partial \gamma} - \frac{c^r}{(1+n)^2} h_t \quad (15)$$

$$(1+\rho) u''(c^r) \frac{\partial c_{t-1}^r}{\partial \gamma} = u''(c^y) \frac{\partial c_t^y}{\partial \gamma} \quad (16)$$

$$u''(c^y) \frac{\partial c_{t-1}^y}{\partial \gamma} = \frac{1}{1+f'(k)} u'(c^y) f''(k) \frac{\partial k_t}{\partial \gamma} + u''(c^y) \frac{\partial c_t^y}{\partial \gamma} \quad (17)$$

Next, from definition (8) it follows that

$$\frac{\partial c_t}{\partial \gamma} = \frac{\partial c_t^y}{\partial \gamma} + \frac{\partial c_{t-1}^r}{\partial \gamma} - \frac{c_t^r}{(1+n)^2} h_t \quad (18)$$

Using eq. (18) in eqs (15)–(17) in order to replace $(\partial c_{t-1}^r/\partial \gamma)$ and $(\partial c_t^y/\partial \gamma)$ by (functions of) $(\partial c_t/\partial \gamma)$ we get eq. (9) where the matrices M and J are defined by

$$J = \begin{bmatrix} \frac{1+r}{1+n} & -\frac{1}{1+n} \\ -\delta(1+r) & 1+\delta \end{bmatrix} \quad M = \begin{bmatrix} 0 & -\frac{k}{1+n} \\ \frac{c^r}{(1+n)^2} & \delta k - \frac{c^r}{(1+n)^2} \end{bmatrix} \quad (19)$$

where $v = (-f''(k)k/f'(k))$ is the (positively defined) elasticity of the production function,

$$\delta = \frac{vr}{\alpha^y k(1+r)(1+n)(1-\varepsilon)} > 0 \quad \text{and} \quad \varepsilon = \frac{\alpha^y}{\alpha^y + (1+n)\alpha^r}.$$

The parameter α^r stands for the absolute rate of risk aversion of a young individual which is defined as $-(u''(c^r)/u'(c^r))$. Analogously, α^y indicates the risk aversion of an old individual. System (9) comprises one predetermined variable (k) and one forward-looking variable (c). The eigenvalues of the Jacobian matrix J are

$$\begin{aligned} \lambda_1 &= \frac{1}{2} \left[2 + \frac{r-n}{1+n} + \delta - \sqrt{\left(\frac{r-n}{1+n} + \delta \right)^2 + 4\delta} \right] \\ \lambda_2 &= \frac{1}{2} \left[2 + \frac{r-n}{1+n} + \delta + \sqrt{\left(\frac{r-n}{1+n} + \delta \right)^2 + 4\delta} \right] \end{aligned} \quad (20)$$

As easily can be checked, one of these eigenvalues lies inside while the other lies outside the unit circle. So the system is saddlepoint stable.